

10/24/16

"Remember no one can make you feel inferior without your consent."-Eleanor Roosevelt

HW: "Chain Rule HW" #1-4  
Test 3 on Friday 11/4

Warm Up:

Suppose someone gave you a present and that it was in a nicely wrapped box, and that box was inside yet another nicely wrapped box.

Simple as it is, think about instructions detailing how you would get to the present, if you want to do so in a polite manner that would impress and demonstrate gratitude.



Ogres have layers!



Onions have layers!

**The Chain Rule:**

If  $y = f(u)$  is a differentiable function of  $u$ , and  $u = g(x)$  is a differentiable function of  $x$ , then  $y = f(g(x))$  is a differentiable function of  $x$  and

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$


$$f(g(x)) \rightarrow \text{Or } f'(g(x)) \cdot g'(x)$$

This looks more complicated than it really is. Basically, the Chain Rule says to **MULTIPLY THE DERIVATIVE OF THE INSIDE FUNCTION BY THE DERIVATIVE OF THE OUTSIDE FUNCTION.**

EX #1: Find the derivative of  $f(x) = (2x+3)^2$  with and without the chain rule.

Inside =  $2x+3$   
Outside =  $x^2$

$$(x)^2 = 2x \cdot 1$$

A.) without chain rule

$$f(x) = (2x+3)^2$$

$$f(x) = 4x^2 + 12x + 9$$

$$f'(x) = 8x + 12$$



B.) with chain rule

$$f(x) = (2x+3)^2$$

$$f'(x) = 2(2x+3)' \cdot (2)$$

$$f'(x) = 8x + 12$$

**The Chain Rule:**  $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$  or  $f'(x) g'(x)$

(Derivative of the outside function)  $\times$  (derivative of the inside function)  
(inside stays as is)

EX #2: Find  $f'(x)$  given  $f(x) = (4 - x^2)^3$

The inside function is  $= 4 - x^2$

The outside function is  $= x^3$

$$f'(x) = 3(4 - x^2)^2(-2x)$$

$$f'(x) = -6x(4 - x^2)^2$$

For each function below, find the derivative.

EX #3:  $f(x) = \sqrt{(x^2 - 1)^3} = ((x^2 - 1)^3)^{\frac{1}{2}} = (x^2 - 1)^{\frac{3}{2}}$

$$f'(x) = \frac{3}{2} (x^2 - 1)^{\frac{1}{2}} \cdot (2x)$$

$$= 3x (x^2 - 1)^{\frac{1}{2}}$$

$$= 3x \sqrt{x^2 - 1}$$

$$\frac{3}{2} \cancel{(2x)} = 3x$$

EX #4:  $f(x) = \frac{-7}{(2x - 3)^2} = -7 (2x - 3)^{-2}$

$$f'(x) = 14 (2x - 3)^{-3} \cdot (2)$$

$$= 28 (2x - 3)^{-3}$$

$$= \frac{28}{(2x - 3)^3}$$

use the chain rule

EX #5:  $y = \frac{1}{2x-3} = 1(2x-3)^{-1}$

$$\begin{aligned} y' &= -1(2x-3)^{-2} \cdot (2) \\ &= -2(2x-3)^{-2} \\ &= \frac{-2}{(2x-3)^2} \end{aligned}$$

Using Quotient rule

$$\begin{aligned} y' &= \frac{(2x-3)(0) - 1(2)}{(2x-3)^2} \\ &= \frac{-2}{(2x-3)^2} \end{aligned}$$

EX #6:  $y = (5 - 4x^2)^{2/3}$

EX #7:  $y = \frac{-2}{\sqrt[3]{6x+3}}$

EX #8:  $y = -3\sqrt{x^2 - 3x - 4} = -3(x^2 - 3x - 4)^{1/2}$

$$\begin{aligned} y' &= -\frac{3}{2} \underbrace{(x^2 - 3x - 4)^{-1/2}}_1 \cdot \underbrace{(2x - 3)}_1 && \begin{array}{l} \text{outside} = -3x^{1/2} \\ \text{inside} = x^2 - 3x - 4 \end{array} \\ &= \frac{-3(2x - 3)}{2(x^2 - 3x - 4)^{1/2}} = \boxed{\frac{-6x + 9}{2\sqrt{x^2 - 3x - 4}}} \end{aligned}$$



EX. #9:  $f(x) = \left( \frac{x-1}{x^2+3} \right)^2$

*This is a quotient so use the quotient rule to get the derivative*

$$f'(x) = 2 \left( \frac{x-1}{x^2+3} \right)' \cdot \left( \frac{(x^2+3)(1) - (x-1)(2x)}{(x^2+3)^2} \right)$$

HW: #7, 10

EX #10:  $f(x) = \frac{x}{\sqrt[3]{x^2+4}}$

7)  $f(x) = \left(\frac{3}{3-x}\right)^2$  Quotient

$\rightarrow f'(x) = 2\left(\frac{3}{3-x}\right)' \cdot \left(\frac{(3-x)(0) - 3(-1)}{(3-x)^2}\right)$

$f'(x) = \left(\frac{6}{3-x}\right) \cdot \left(\frac{3}{(3-x)^2}\right)$

$f'(x) = \frac{18}{(3-x)^3}$

10)  $h(x) = \boxed{x^3}^{\text{1st}} \boxed{(5-3x)^4}^{\text{2nd}}$  ← Start with product rule.

$h'(x) = \boxed{x^3}^{\text{1st}} \boxed{-12(5-3x)^3}^{\text{dsecond}} + \boxed{(5-3x)^4}^{\text{2nd}} \boxed{3x^2}^{\text{dfirst}}$

$h'(x) = -12x^3(5-3x)^3 + (5-3x)^4 \cdot 3x^2$

Derivatives:

$1^{\text{st}} = 3x^2$

$2^{\text{nd}} = 4(5-3x)^3(-3)$   
 $= -12(5-3x)^3$

$$16) \quad y = \sin(3x)$$

$$y' = \cos(\underline{3x}) \cdot (3)$$

$$y' = 3 \cos(3x)$$

$$17) \quad y = \cos^2(x) = (\cos(x))^2$$

$$y' = 2 (\cos(x))' \cdot (-\sin(x))$$

HW: #22, 24

22)  $y = \sqrt{x^2 + 3x + 6}$  @ (2, 4)

$y = (x^2 + 3x + 6)^{\frac{1}{2}}$

$y' = \frac{1}{2} (x^2 + 3x + 6)^{-\frac{1}{2}} \cdot (2x + 3)$

24) Hint:  $\frac{\pi}{3} = 60^\circ$